# Filtering
# Filtering properties
# Correlation & convolution

# Image Filtering

Compute function of local neighborhood
at each position:

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Really important!

- Enhance images
  - Denoise, resize, increase contrast, etc.
- Extract information from images
  - Texture, edges, distinctive points, etc.
- Detect patterns
  - Template matching

# Linear Filters

**Linearity:**

$$\mathtt{imfilter(I,\ f_1\ +\ f_2)\ =}$$
$$\mathtt{imfilter(I,f_1)\ +\ imfilter(I,f_2)}$$

**Shift/translation invariance:**

Same behavior given intensities regardless of pixel location m,n

$$\mathtt{imfilter(I,shift(f))\ =}$$
$$\mathtt{shift(imfilter(I,f))}$$

**Any linear, shift-invariant operator can be represented as a convolution.**

# Correlation and Convolution

**Definition**

2D correlation

$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

```
e.g., h = scipy.signal.correlate2d(f,I)
```

2D convolution

$$h[m,n]=\sum_{k,l} f[k,l]\, I[m-k,n-l]$$

```
e.g., h = scipy.signal.convolve2d(f,I)
```

Convolution is the same as correlation with a 180° rotated filter kernel.
Correlation and convolution are identical when the filter kernel is rotationally symmetric*.

James Hays

* Symmetric in the geometric sense, not in the matrix linear algebra sense

# Cross-correlation

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

w

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|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

f

$$1*1 + 2*2 + 3*3 + 4*4 + 5*5 + 6*6 + 7*7 + 8*8 + 9*9$$

# Convolution

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

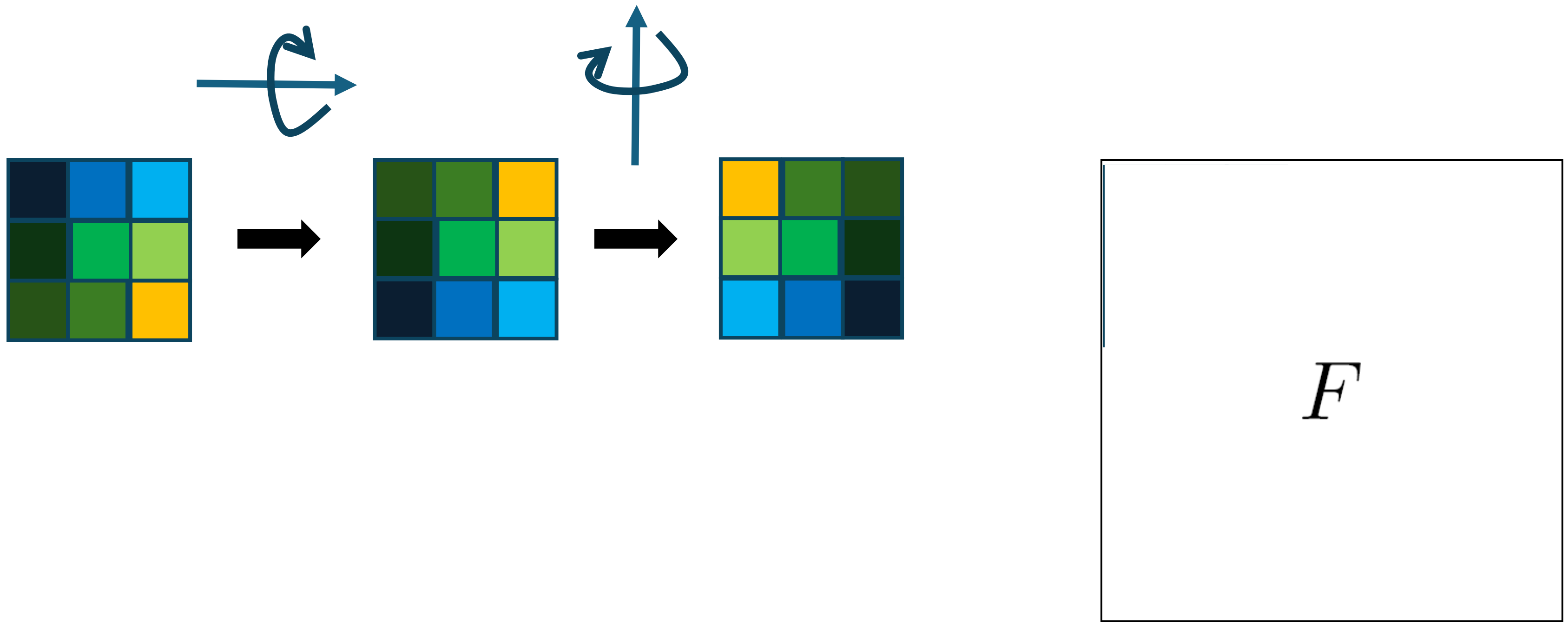
w

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

f

$$1*9 + 2*8 + 3*7 + 4*6 + 5*5 + 6*4 + 7*3 + 8*2 + 9*1$$

# Convolution



# Convolution Properties

Commutative: $a * b = b * a$

- Conceptually no difference between filter and signal
- But filtering implementations might break this equality, e.g., image edges
- Correlation is _not_ commutative (rotation effect) – produces rotated version of output.

Associative: $a * (b * c) = (a * b) * c$

- Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
- This is equivalent to applying one filter: a * $(b_1 * b_2 * b_3)$ -> computationally faster
- Correlation is _not_ associative (rotation effect)

Distributes over addition: $a * (b + c) = (a * b) + (a * c)$

Scalars factor out: $ka * b = a * kb = k (a * b)$

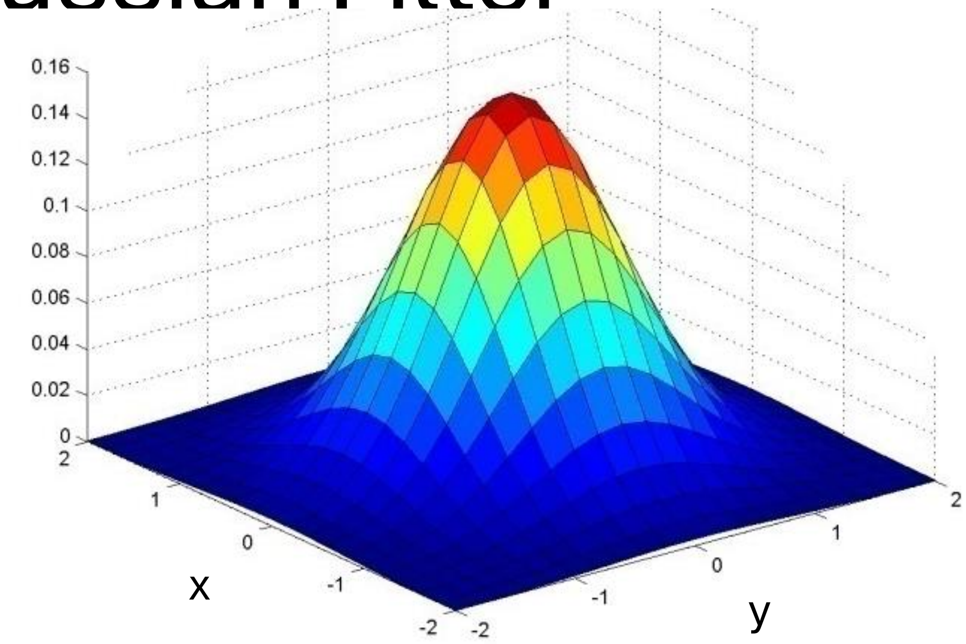
Identity: $a * e = a$ when $e = [0, 0, 1, 0, 0]$,

# Convolution in Convolutional Neural Networks

- Convolution is the basic operation in CNNs

- Learning convolution kernels allows us to learn which `features’ provide useful information in images.

# Gaussian Filter



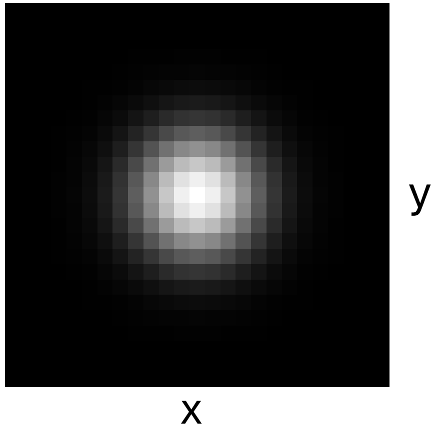

x

| | | | | |
|---|---|---|---|---|
| 0.003 | 0.013 | 0.022 | 0.013 | 0.003 |
| 0.013 | 0.059 | 0.097 | 0.059 | 0.013 |
| 0.022 | 0.097 | 0.159 | 0.097 | 0.022 |
| 0.013 | 0.059 | 0.097 | 0.059 | 0.013 |
| 0.003 | 0.013 | 0.022 | 0.013 | 0.003 |

y

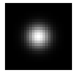
Kernel size 5 x 5,
Standard deviation $\sigma$ = 1

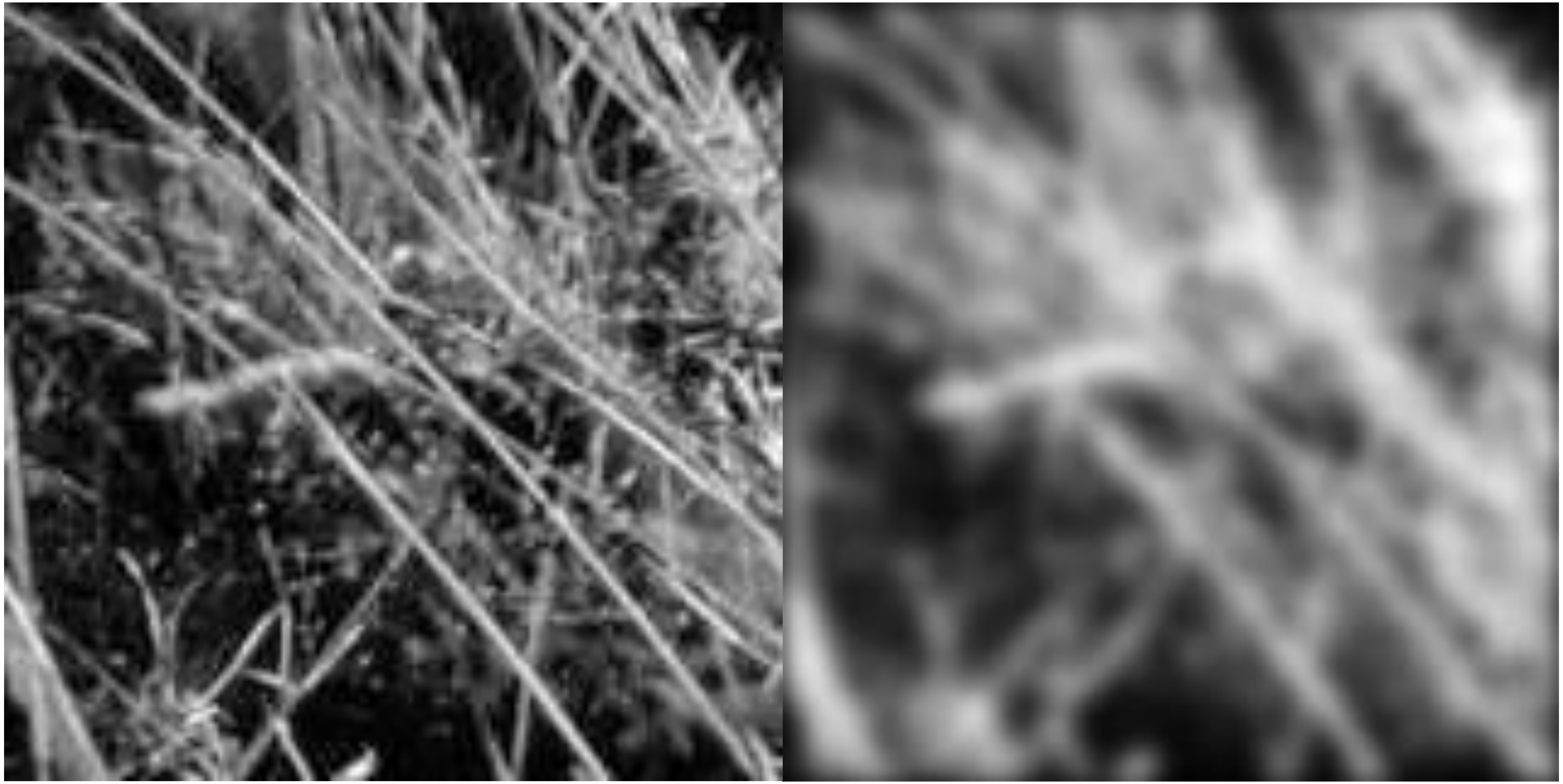
Viewed from top




$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

# Smoothing with Gaussian Filter

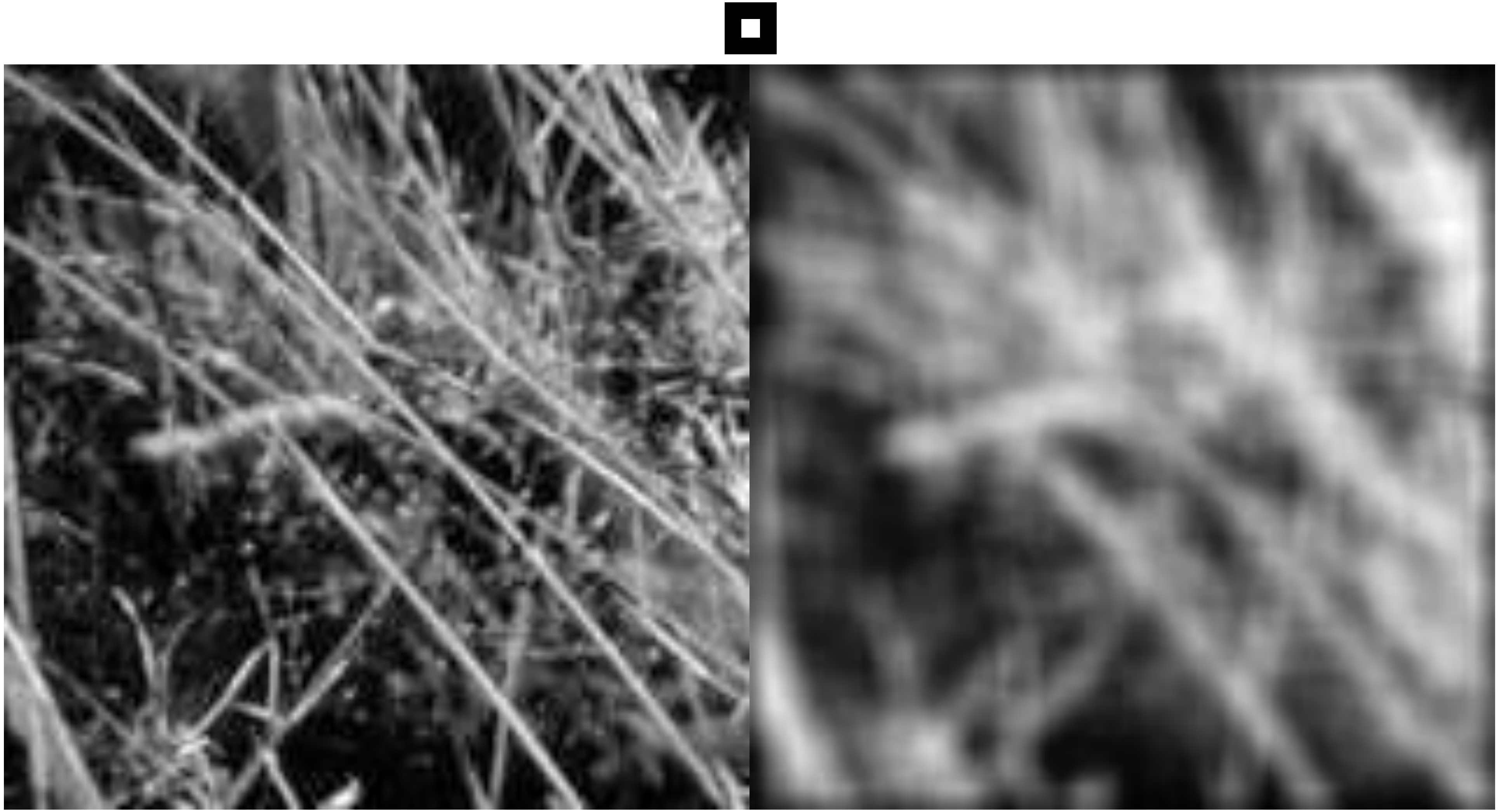




James Hays

# Smoothing with Box Filter

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



David Lowe

# Gaussian Filter Properties

Gaussian convolved with Gaussian…

…is another Gaussian

- So can smooth with small-width kernel, repeat, and get same result as larger-width kernel
- Convolving twice with Gaussian kernel of width $\sigma$ is same as convolving once with kernel of width $\sigma\sqrt{2}$

*Separable* kernel

- Factors into product of two 1D Gaussians

# Separability of the Gaussian Filter

$$
\begin{aligned}
G_\sigma(x, y) &= \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}} \\
&= \left( \frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}} \right) \left( \frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}} \right)
\end{aligned}
$$

The 2D Gaussian can be expressed as the product of two functions, one a function of $x$ and the other a function of $y$

In this case, the two functions are the (identical) 1D Gaussian

# Separability Example

2D convolution
(center location only)

=

$$
\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix}
\begin{array}{l} = 2 + 6 + 3 = 11 \\ = 6 + 20 + 10 = 36 \\ = 4 + 8 + 6 = \underline{18} \\ \qquad\qquad\qquad\;\; 65 \end{array}
$$

$$
\begin{bmatrix} \; & \; & \; \\ & 65 & \\ & & \end{bmatrix}
$$

# Separability Example

2D convolution
(center location only)

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix} \quad \begin{aligned} &= 2 + 6 + 3 = 11\\ &= 6 + 20 + 10 = 36\\ &= 4 + 8 + 6 = 18\\ \hline &\quad 65 \end{aligned} \quad = \begin{bmatrix}\ &\ &\ \\ \ &65&\ \\ \ &\ &\ \end{bmatrix}$$

The filter factors
into a product of 1D
filters:

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix} \times \begin{bmatrix}1&2&1\end{bmatrix}$$

Perform convolution
along rows:

$$\begin{bmatrix}1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix} = \begin{bmatrix}\ &11&\ \\ \ &18&\ \\ \ &18&\ \end{bmatrix}$$

Followed by convolution
along the remaining column:

$$\begin{bmatrix}1\\2\\1\end{bmatrix} * \begin{bmatrix}\ &11&\ \\ \ &18&\ \\ \ &18&\ \end{bmatrix} = \begin{bmatrix}\ &\ &\ \\ \ &65&\ \\ \ &\ &\ \end{bmatrix}$$

Kristen Grauman

# Separability Use

MxN image, PxQ filter

- 2D convolution: ~MNPQ multiply-adds
- Separable 2D: ~MN(P+Q) multiply-adds

Speed up = PQ/(P+Q)

9x9 filter = ~4.5x faster

# Example: Box Filter

$$f[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)
- Why does it sum to one?

# Image Filtering

$f[\cdot\,,\cdot\,]\ \frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$I[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

$m = 1, n = 1$
$k, l = [-1,0,1]$

Steve Seitz

# Image Filtering

$f[\cdot,\cdot]\ \frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
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$I[.,.]$

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

$m = 1, n = 1$
$k,l = [-1,0,1]$

Steve Seitz

# Image Filtering

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$I[.,.]$

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
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$$h[m,n]=\sum_{k,l} f[k,l]\,I[m+k,n+l]$$

$$m = 2, n = 1$$
$$k, l = [-1,0,1]$$

Steve Seitz

# Image Filtering

$$f[\cdot\,,\cdot\,]\;\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$I[.,.]$

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|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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$$h[m,n]=\sum_{k,l} f[k,l]\,I[m+k,n+l]$$

$$m = 3, n = 1$$
$$k,l = [-1,0,1]$$

Steve Seitz

# Image Filtering

$f[\cdot\,,\cdot\,]$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$I[.,.]$

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
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| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

$m = 4, n = 1$
$k,l = [-1,0,1]$

Steve Seitz

# Image Filtering

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$I[.,.]$

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|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
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$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

$$m = 5, n = 1$$
$$k,l = [-1,0,1]$$

Steve Seitz

# Image Filtering

$$f[\cdot,\cdot] \; \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$I[.,.]$

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|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
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$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

$$m = 4, n = 6$$
$$k,l = [-1,0,1]$$

Steve Seitz

# Image Filtering

$$f[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$I[.,.]$

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|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

$$m = 6, n = 4$$
$$k, l = [-1,0,1]$$

Steve Seitz

# Image Filtering

$$f[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$I[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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$h[.,.]$

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| | 0 | 30 | 60 | 90 | 90 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 10 | 20 | 30 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | |
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$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

Steve Seitz

1. $$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. $$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

3. $$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

4. $$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

David Lowe

# 1. Practice with linear filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

?

# 1. Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Filtered
(no change)

David Lowe

# 2. Practice with linear filters

Original

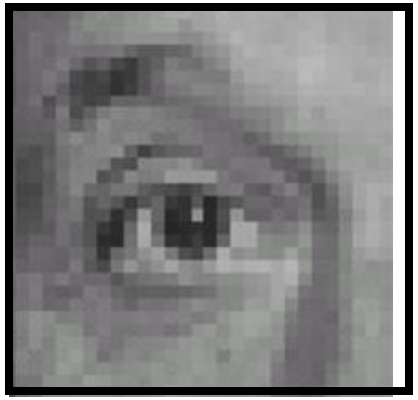
| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

?

# 2. Practice with linear filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |



Shifted left
By 1 pixel

David Lowe

# Sobel Filter

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

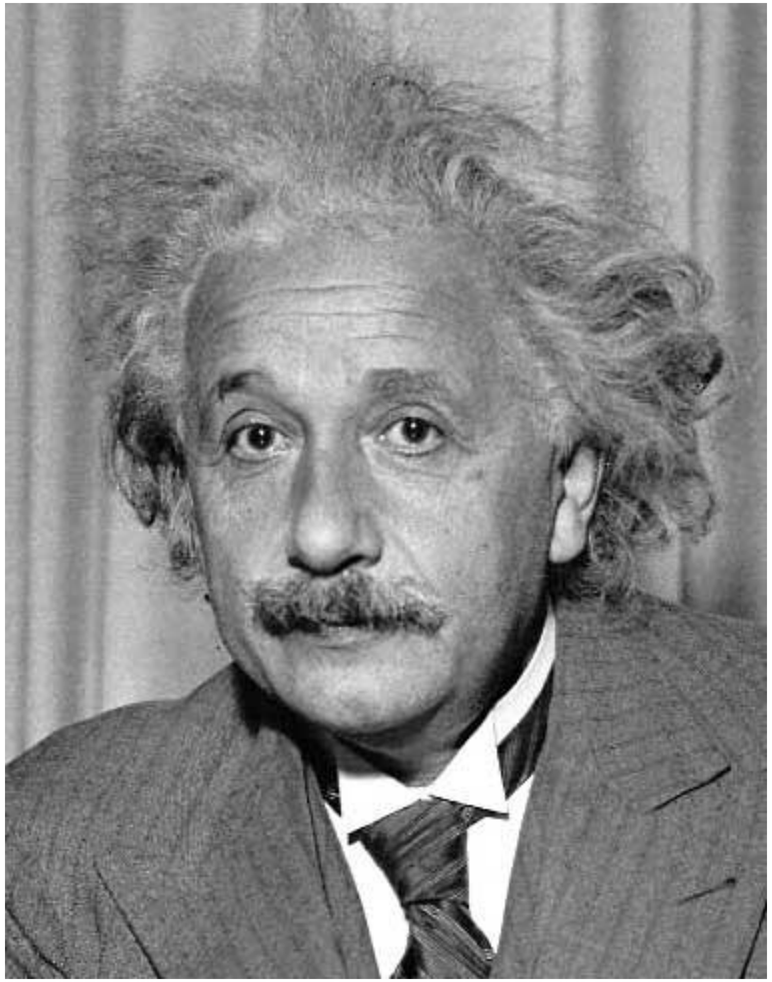
Sobel

- What is the 1 2 1 pattern?
  - Binomial distribution or “triangle filter”
  - Discrete approximate to Gaussian; fast for integer mathematics
  - Smooths along edge

- What happens to negative numbers?

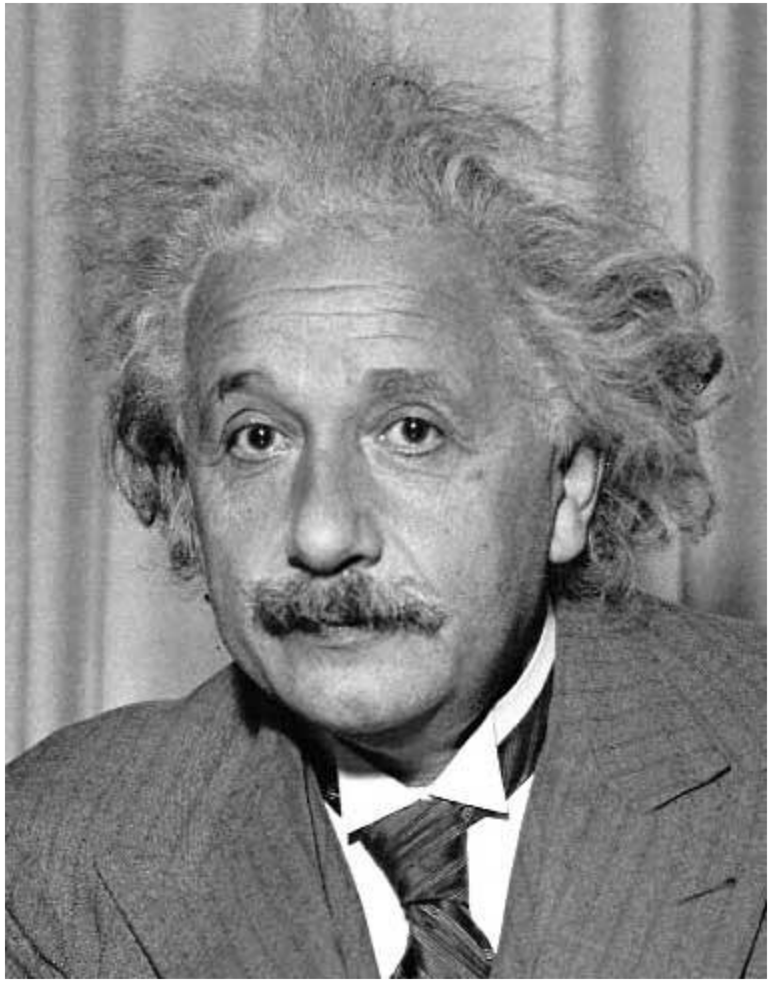
- For visualization:
  - Shift image + 0.5
  - If gradients are small, scale edge response

# 3. Practice with linear filters (Sobel Filter)



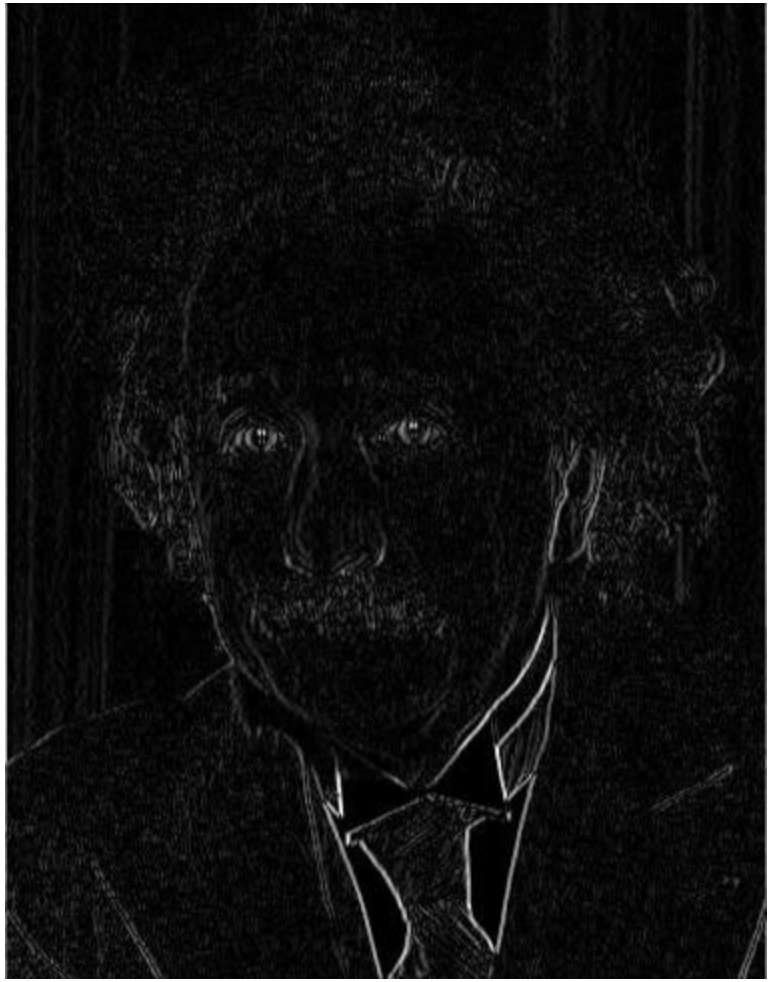
| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

# 3. Practice with linear filters (Sobel Filter)



|  |  |  |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

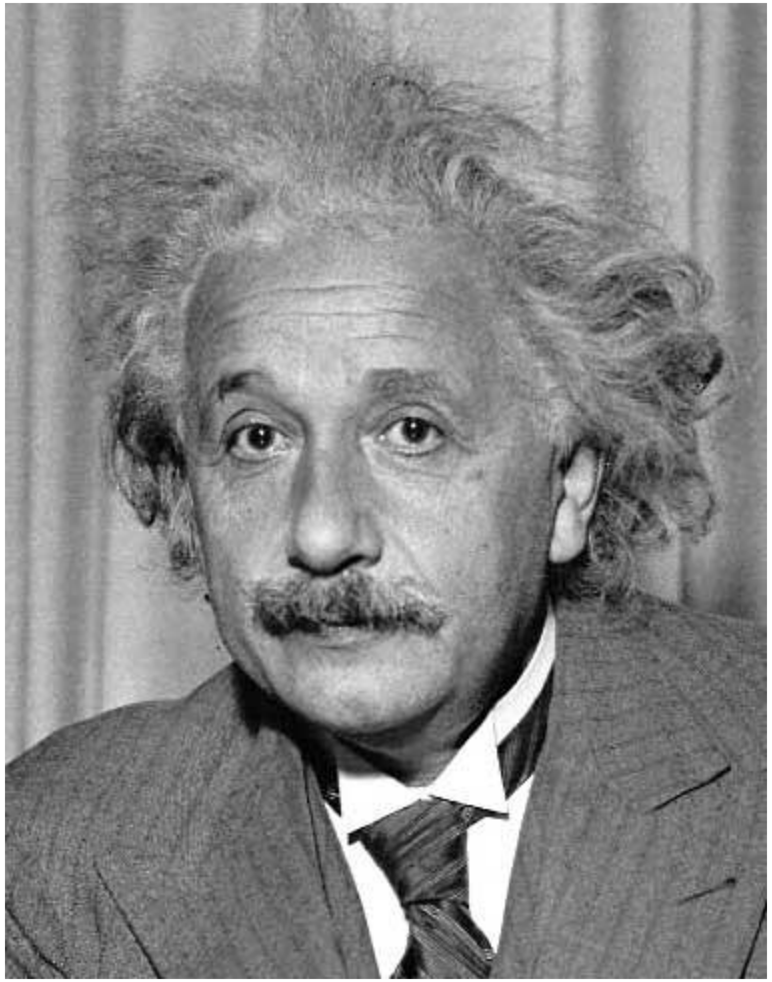
Sobel



Vertical Edge
(absolute value)

David Lowe

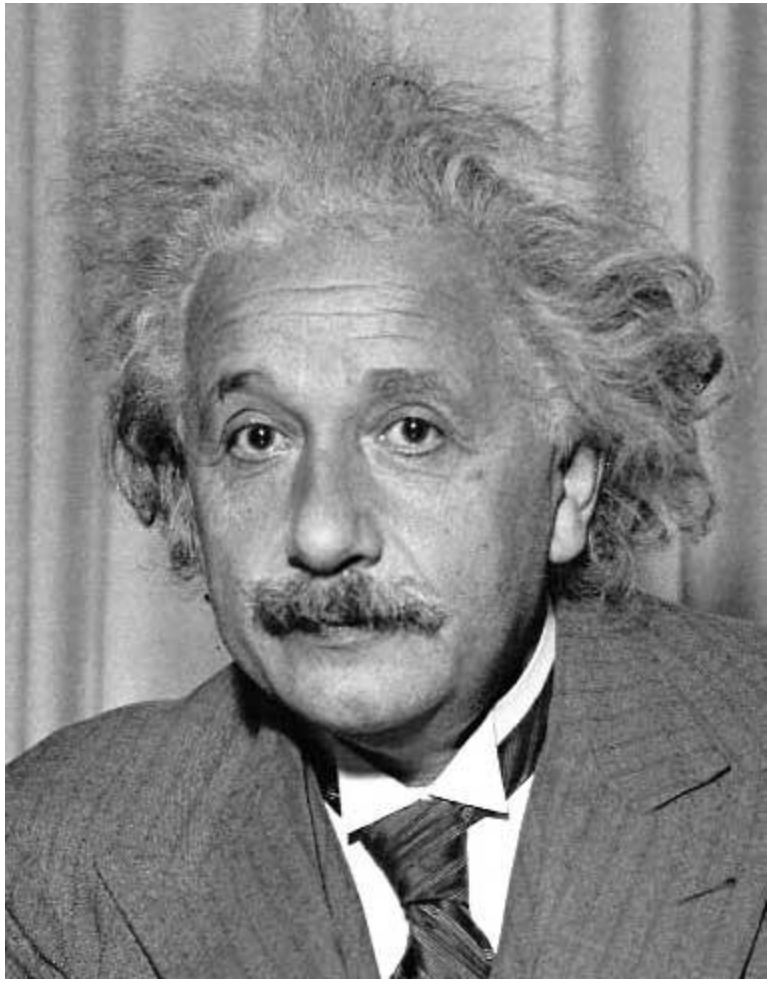
# 3. Practice with linear filters (Sobel Filter)



| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

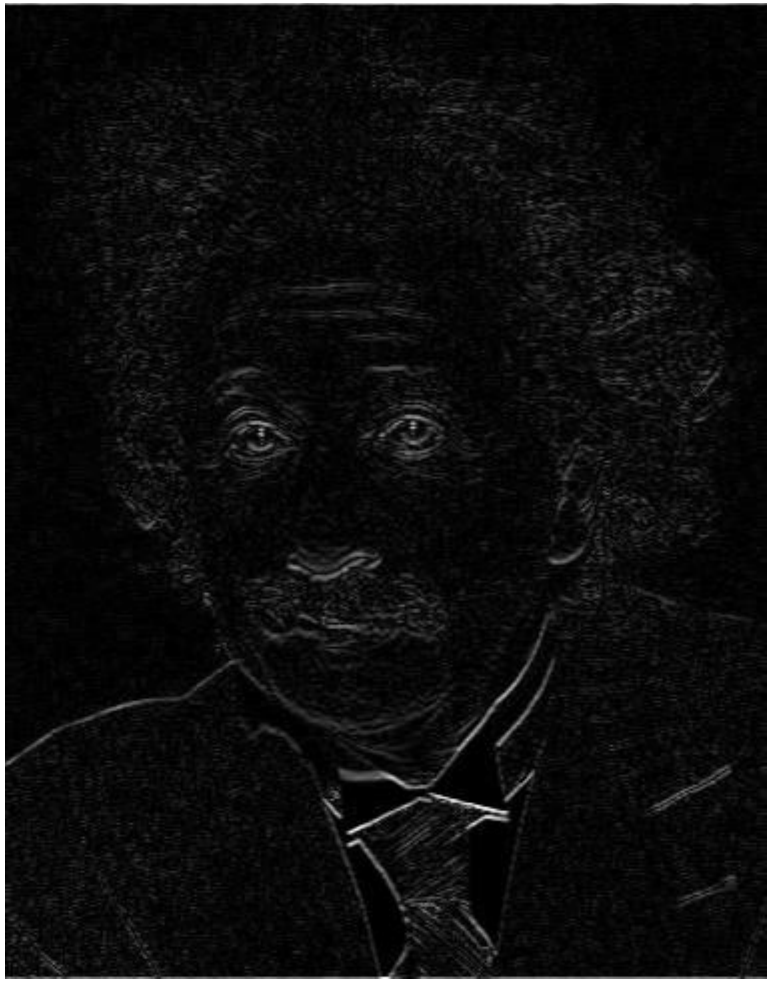
David Lowe

# 3. Practice with linear filters (Sobel Filter)



| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

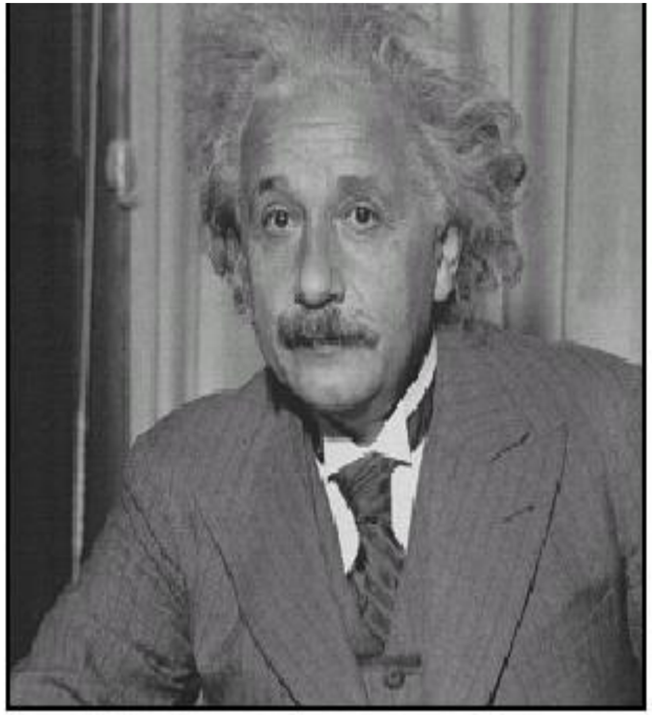
Sobel



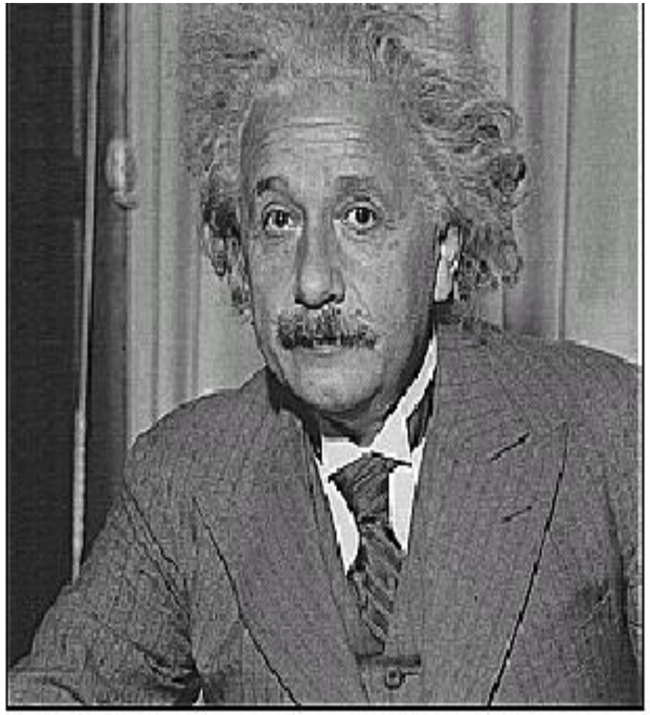
Horizontal Edge
(absolute value)

David Lowe

# Sharpening



before

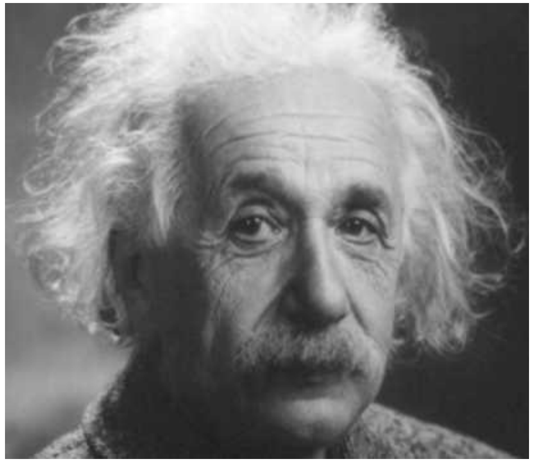


after

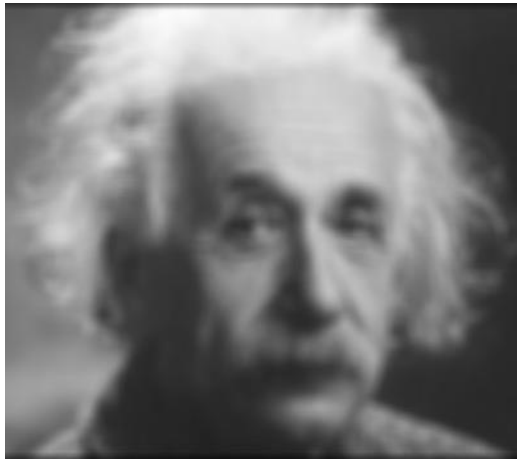
Source: D. Lowe

# Sharpening

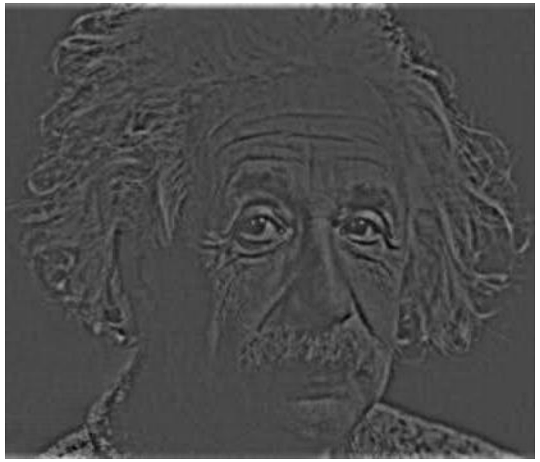
- What does blurring take away?



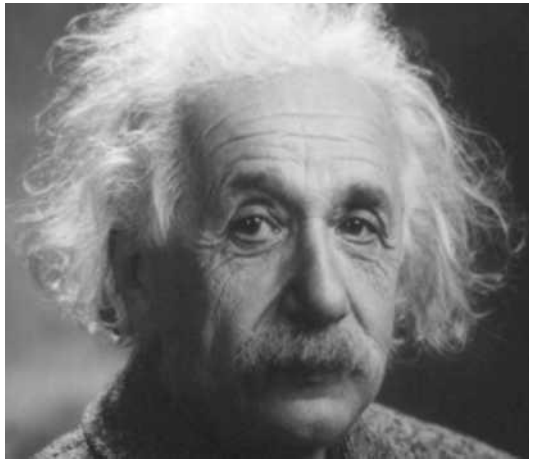
–



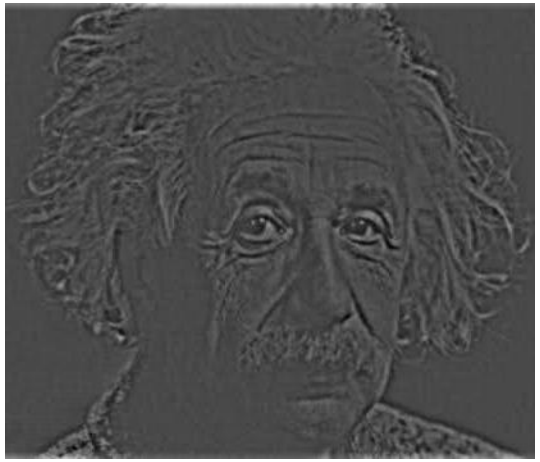
=



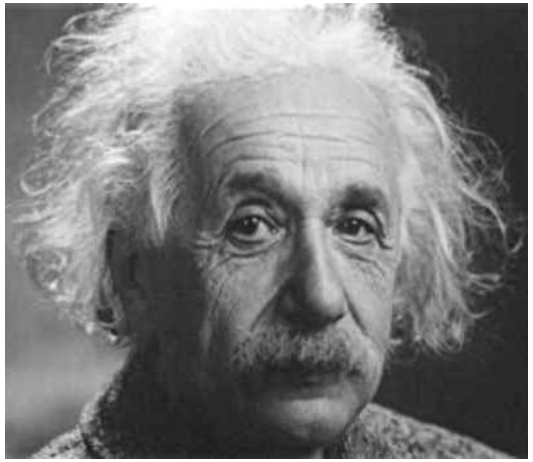
Let’s add it back:



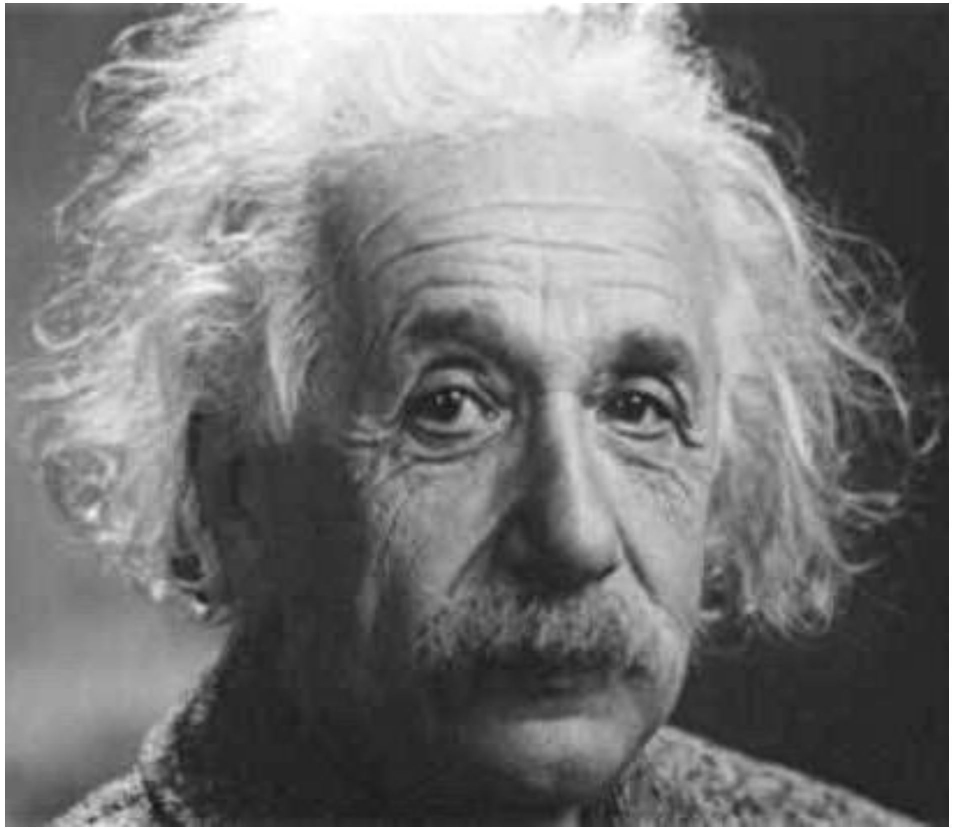
+ α



=

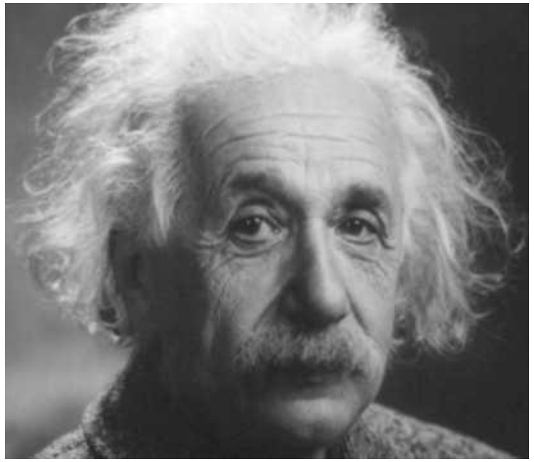


# Sharpening

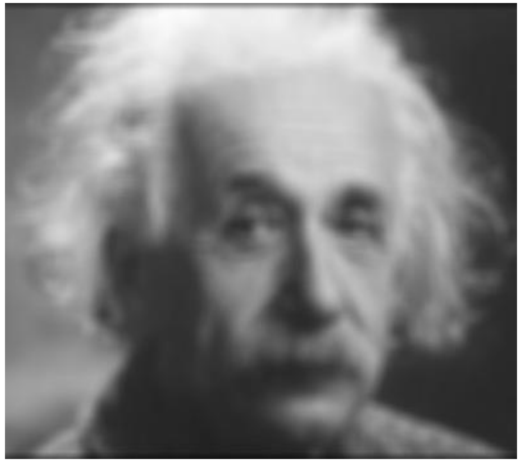


# Sharpening

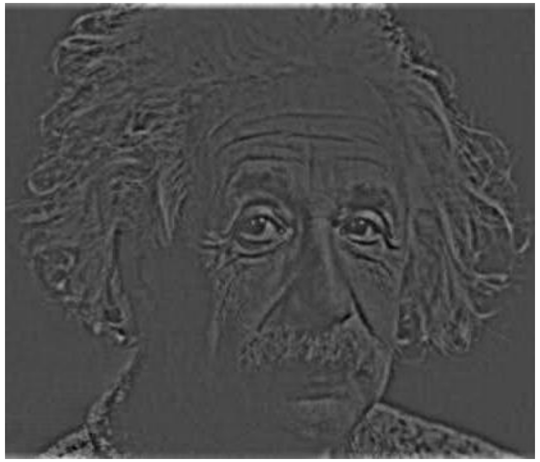
- What does blurring take away?



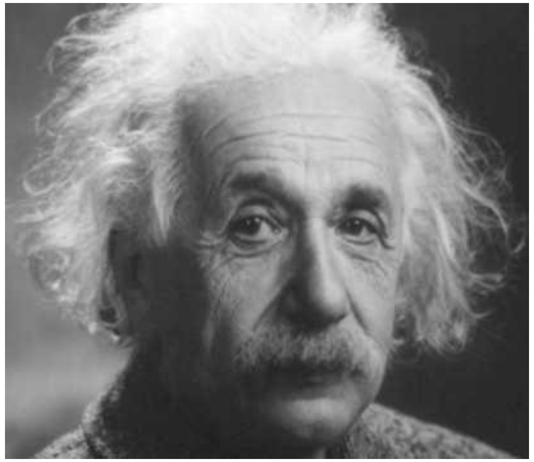
–



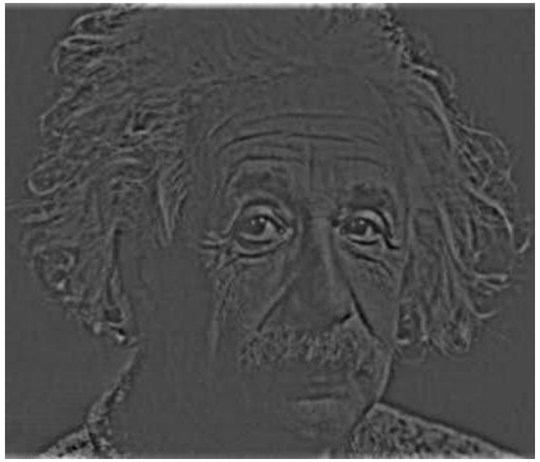
=



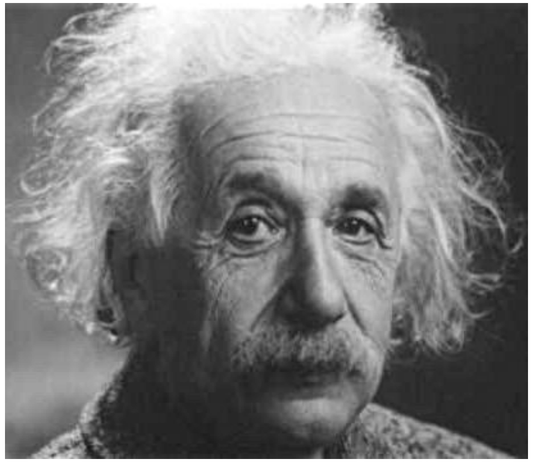
Let’s add it back:



+ α



=



# Sharpening

$$
\begin{aligned}
f_{sharp} &= f + \alpha(f - f_{blur}) \\
&= (1+\alpha)f - \alpha f_{blur} \\
&= (1+\alpha)(w * f) - \alpha(v * f)
\end{aligned}
$$

$$
w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad v = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}
$$

$$
= ((1+\alpha)w - \alpha v) * f
$$

# Sharpening filter

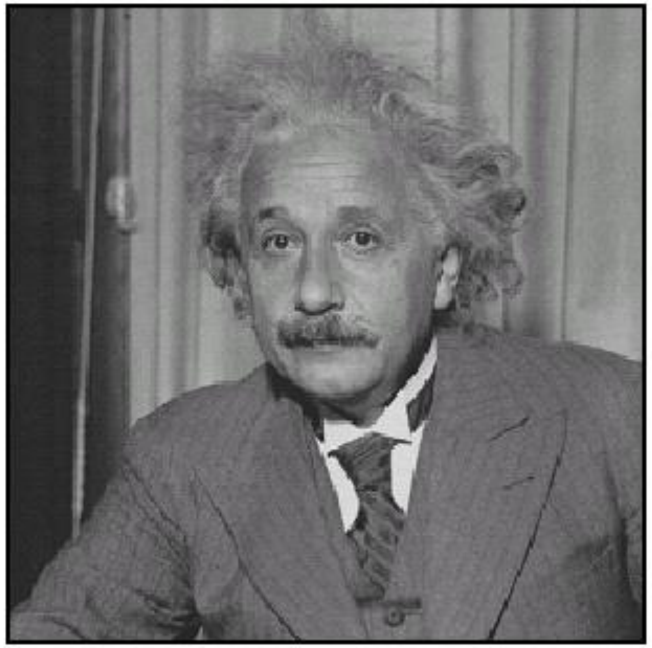
Original

$$* \left( \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$

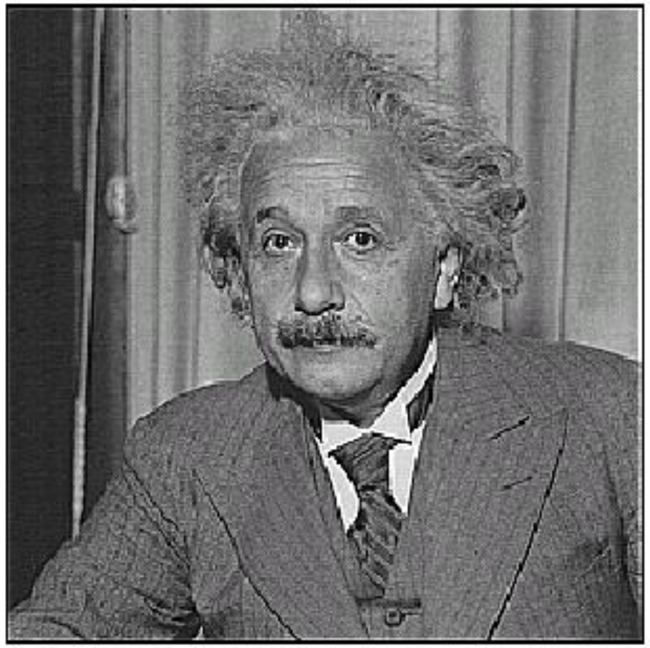
**Sharpening filter**
(accentuates edges)

Source: D. Lowe

# 4. Practice with linear filters



**before**



**after**

David Lowe

# Filters in Practice

What about near the edge?

- The filter window falls off the edge
- Need to extrapolate
- methods:
  - clip filter (black)
  - wrap around
  - copy edge
  - reflect across edge

# Filters in Practice

What about near the edge?

- The filter window falls off the edge
- Need to extrapolate
- methods:
  - clip filter (black)
  - wrap around
  - copy edge
  - reflect across edge

# Filters in Practice

What about near the edge?

- The filter window falls off the edge of the image
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- methods:
  - clip filter (black)

# Filters in Practice

What about near the edge?

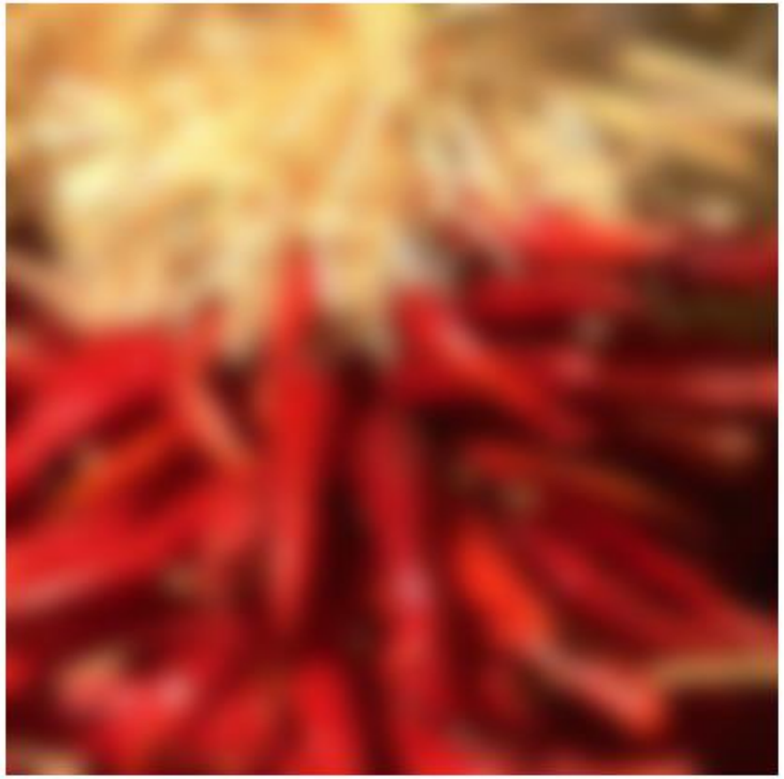
- The filter window falls off the edge of the image
- Need to extrapolate
- methods:
  - wrap around

S. Marschner

# Filters in Practice

What about near the edge?

- The filter window falls off the edge of the image
- Need to extrapolate
- methods:
  - copy edge



# Filters in Practice

What about near the edge?

- The filter window falls off the edge of the image
- Need to extrapolate
- methods:
  - reflect across edge

S. Marschner

# Non-linear Filters

# Non-Linear Filter: Median Filter

- Operates over a window by selecting the median intensity in the window.
- ‘Rank’ filter as based on ordering of gray levels
    - E.G., min, max, range filters

# Median Filter

$I[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
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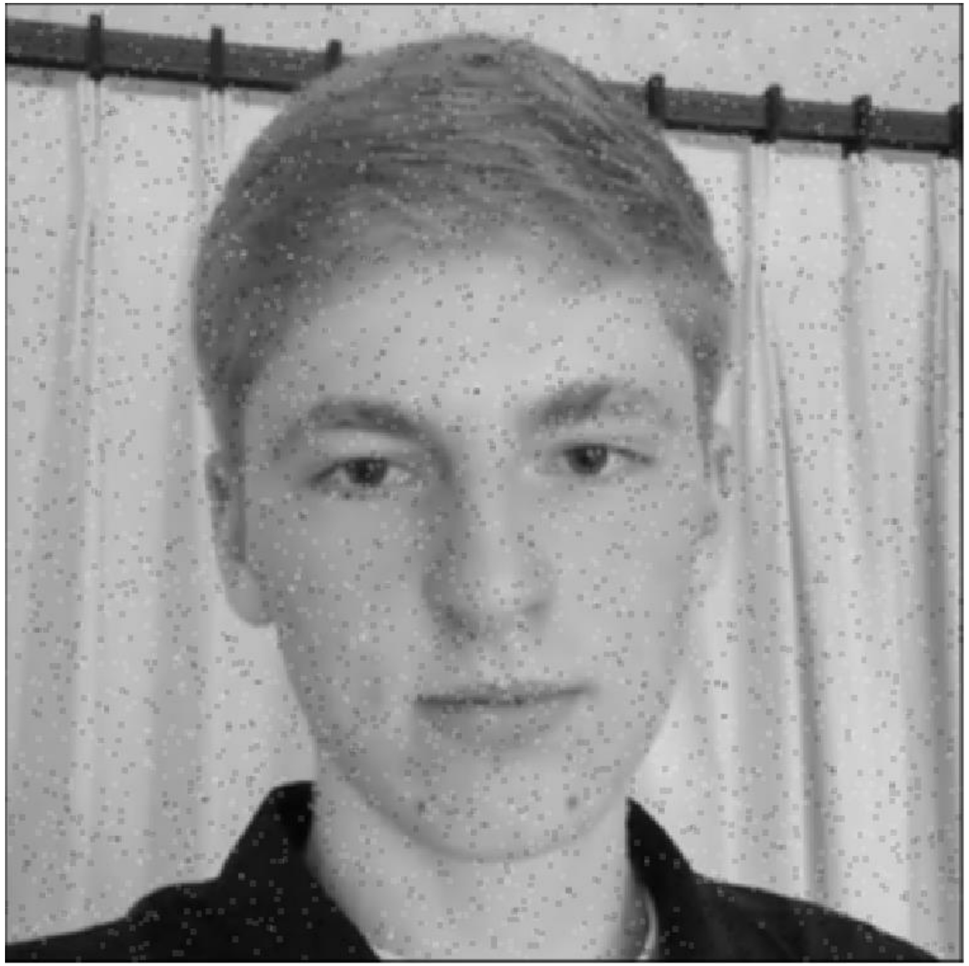
| | | | | | | | | | |

Steve Seitz

# Salt and Pepper Noise

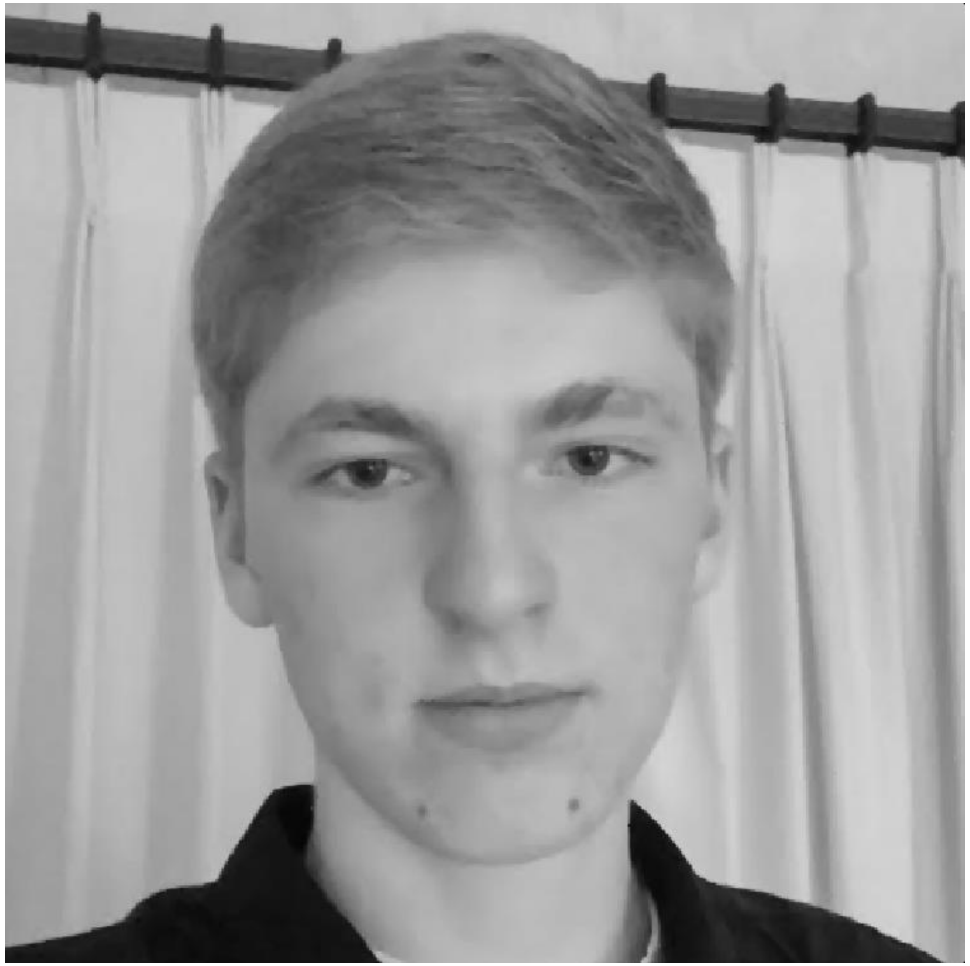
# 3 x 3 Mean Filter



# 11 x 11 Mean Filter

# Salt and Pepper Noise

# 3 x 3 Median Filter



# 11 x 11 Median Filter

# Median filters

- Operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

*Interpretation:* Median filtering is sorting.

# Question

- Consider the following image. what will be the new value of the pixel (2,2) of smoothing is done using a 3 x 3 kernel.

$$I = \begin{bmatrix} 10 & 20 & 30 & 40 & 50 \\ 15 & 25 & 35 & 45 & 55 \\ 20 & 30 & 40 & 50 & 60 \\ 25 & 35 & 45 & 55 & 65 \\ 30 & 40 & 50 & 60 & 70 \end{bmatrix}$$

# Question

- Mean Filter (Simple average).
- Average Weighted Filter
- Let’s assume a common **weighted kernel** (e.g., Gaussian-like):

$$K = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

- Median Filter
- Min Filter
- Max Filter

# Question

- Define the neighborhood for pixel (2,2).
- Assuming **0-based indexing**, pixel **(2,2)** corresponds to the value **40** (third row, third column).
- The **3×3 neighborhood** around (2,2) is:

$$N = \begin{bmatrix} 25 & 35 & 45 \\ 30 & 40 & 50 \\ 35 & 45 & 55 \end{bmatrix}$$

# Edge Detection

https://towardsdatascience.com/edge-detection-in-python-a3c263a13e03

# Edge Detection

**Definition**

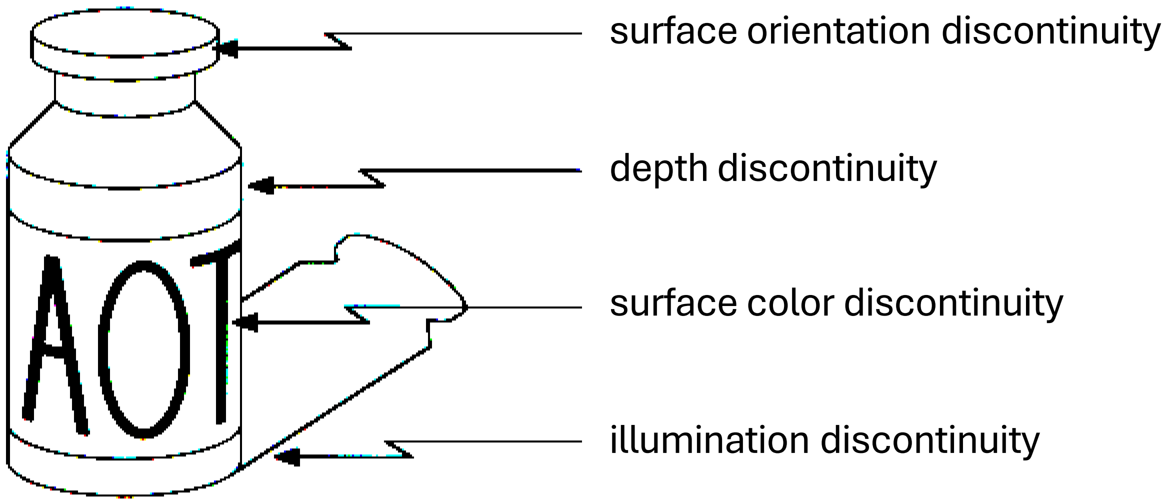
The process of identifying parts of a digital image with sharp changes (discontinuities) in image intensity.

Beginning to extract information.

Helpful to

- Recognize objects
- Reconstruct scenes
- Edit images (artistically)

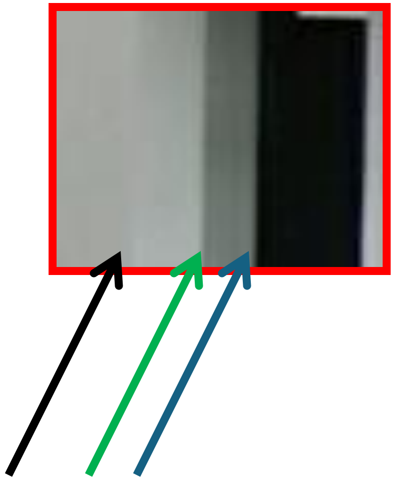
# Causes of Edges




Edges are caused by a variety of factors

# Closer Look at Edges

# Closer Look at Edges



# Closer Look at Edges

# Closer Look at Edges